3–3 The Angle Addition Postulate

In the following activity, you will learn about the Angle Addition Postulate.

Hands-On Geometry

Materials: straightedge   protractor
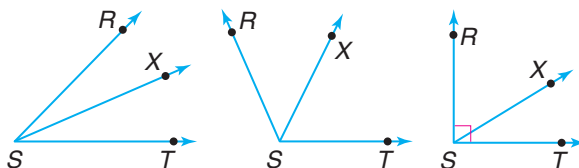
Step 1 Draw an acute, an obtuse, and a right angle. Label each angle *RST*.
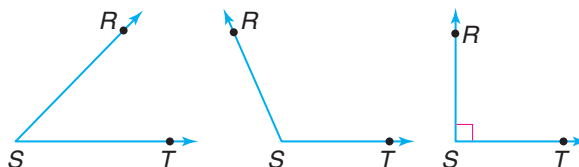
Step 2 Draw and label a point *X* in the interior of each angle. Then draw $\overrightarrow{SX}$.

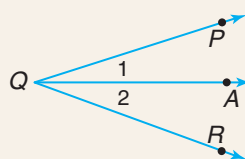**Step 3** For each angle, find $m\angle RSX$, $m\angle XST$, and $m\angle RST$.

Try These
1. For each angle, how does the sum of $m\angle RSX$ and $m\angle XST$ compare to $m\angle RST$?
2. **Make a conjecture** about the relationship between the two smaller angles and the larger angle.

The activity above leads to the following postulate.

Postulate 3–3 **Angle Addition Postulate** **(A–A Postulate)**	**Words:** For any angle *PQR*, if *A* is in the interior of $\angle PQR$, then $m\angle PQA + m\angle AQR = m\angle PQR$. **Model:** **Symbols:** $m\angle 1 + m\angle 2 = m\angle PQR$

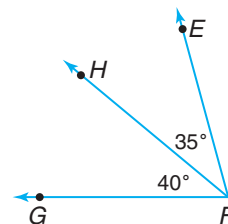There are two equations that can be derived using Postulate 3–3.

$m\angle 1 = m\angle PQR - m\angle 2$ *These equations are true no matter where*
$m\angle 2 = m\angle PQR - m\angle 1$ *A is located in the interior of $\angle PQR$.*

CONTENTS

1 If $m\angle EFH = 35$ and $m\angle HFG = 40$, find $m\angle EFG$.

$m\angle EFG = m\angle EFH + m\angle HFG$
$\quad\quad\quad = \quad 35 \quad + \quad 40 \quad$ *Substitution*
$\quad\quad\quad = 75$
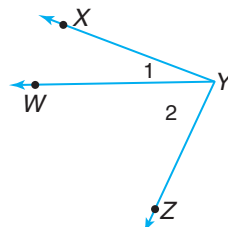
So, $m\angle EFG = 75$.

2 Find $m\angle 2$ if $m\angle XYZ = 86$ and $m\angle 1 = 22$.

$m\angle 2 = m\angle XYZ - m\angle 1$
$\quad\quad = \quad 86 \quad - \quad 22 \quad$ *Substitution*
$\quad\quad = 64$

So, $m\angle 2 = 64$.

Algebra Link

3 Find $m\angle ABC$ and $m\angle CBD$ if $m\angle ABD = 120$.
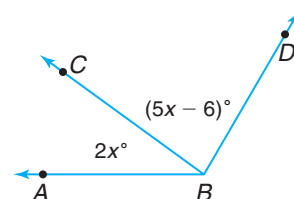
$m\angle ABC + m\angle CBD = m\angle ABD \quad$ *Postulate 3–3*
$\quad\quad\quad 2x + (5x - 6) = 120 \quad$ *Substitution*
$\quad\quad\quad\quad 7x \quad\quad - 6 = 120 \quad$ *Combine like terms.*
$\quad\quad 7x - 6 + 6 = 120 + 6 \quad$ *Add 6 to each side.*
$\quad\quad\quad\quad\quad 7x = 126$
$\quad\quad\quad\quad \dfrac{7x}{7} = \dfrac{126}{7} \quad\quad$ *Divide each side by 7.*
$\quad\quad\quad\quad\quad x = 18$

To find $m\angle ABC$ and $m\angle CBD$, replace x with 18 in each expression.

$m\angle ABC = 2x \quad\quad\quad\quad\quad m\angle CBD = 5x - 6$
$\quad\quad\quad = 2(18) \quad x = 18 \quad\quad\quad\quad = 5(18) - 6 \quad x = 18$
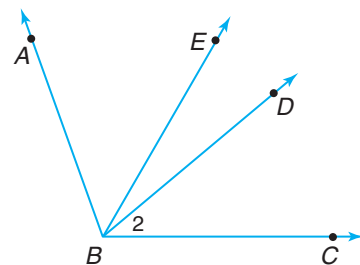$\quad\quad\quad = 36 \quad\quad\quad\quad\quad\quad\quad\quad = 90 - 6 \text{ or } 84$

So, $m\angle ABC = 36$ and $m\angle CBD = 84$.
Check: Is the sum of the measures 120?

Algebra Review

Solving Multi-Step Equations, p. 723

Your Turn

a. Find $m\angle ABC$ if $m\angle ABD = 70$ and $m\angle DBC = 43$.

b. If $m\angle EBC = 55$ and $m\angle EBD = 20$, find $m\angle 2$.

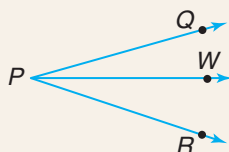c. Find $m\angle ABD$ if $m\angle ABC = 110$ and $m\angle 2 = 36$.

CONTENTS

Just as every segment has a midpoint that bisects the segment, every angle has a ray that bisects the angle. This ray is called an **angle bisector**.

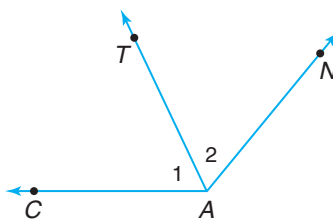| **Definition of an Angle Bisector**	**Words:** The bisector of an angle is the ray with its endpoint at the vertex of the angle, extending into the interior of the angle. The bisector separates the angle into two angles of equal measure.
	Model:
	Symbols: $\overrightarrow{PW}$ is the bisector of $\angle P$. $m\angle QPW = m\angle WPR$

Example ④ If $\overrightarrow{AT}$ bisects $\angle CAN$ and $m\angle CAN = 130$, find $m\angle 1$ and $m\angle 2$.

Since $\overrightarrow{AT}$ bisects $\angle CAN$, $m\angle 1 = m\angle 2$.

$m\angle 1 + m\angle 2 = m\angle CAN$ *Postulate 3–3*

$m\angle 1 + m\angle 2 = 130$ *Replace $m\angle CAN$ with 130.*

$m\angle 1 + m\angle 1 = 130$ *Replace $m\angle 2$ with $m\angle 1$.*

$2(m\angle 1) = 130$ *Combine like terms.*
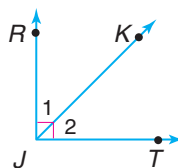
$\dfrac{2(m\angle 1)}{2} = \dfrac{130}{2}$ *Divide each side by 2.*

$m\angle 1 = 65$

Since $m\angle 1 = m\angle 2$, $m\angle 2 = 65$.

Your Turn

d. If $\overrightarrow{JK}$ bisects $\angle RJT$ and $\angle RJT$ is a right angle, find $m\angle 1$ and $m\angle 2$.
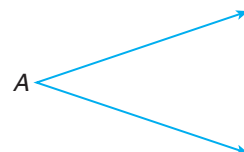
CONTENTS

The angle bisector of a given angle can be constructed using the following procedure.
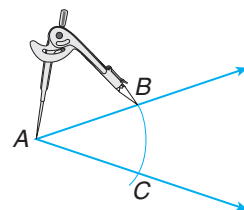
Hands-On Geometry
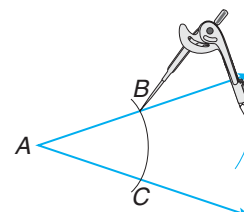### Construction

Materials: compass straightedge

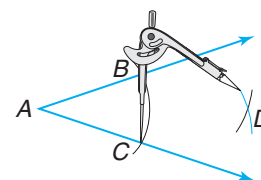**Step 1** Draw an angle like $\angle A$ on your paper.

A

Step 2 Place a compass at point A and draw a large arc that intersects both sides of $\angle A$. Label the points of intersection B and C.
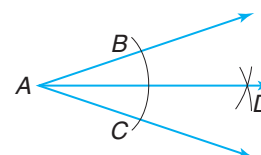
A *B* *C*

Step 3 With the compass at point B, draw an arc in the interior of $\angle A$.

A *B* *C*

Step 4 Keeping the same compass setting, place the compass at point C. Draw an arc that intersects the arc drawn in Step 3. Label the point of intersection D.

A *B* *C* *D*

Step 5 Draw $\overrightarrow{AD}$.

A *B* *C* *D*

Try These

1. How does $m\angle BAD$ compare to $m\angle DAC$?
2. Name the bisector of $\angle BAC$.
3. Draw an angle like $\angle Y$ on your paper. Then construct the angle bisector of $\angle Y$.

Y

◀ CONTENTS ▶
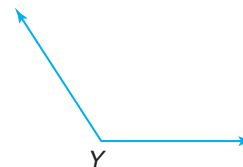
Check for Understanding

1. **State** the Angle Addition Postulate in your own words.

2. **Draw** an acute angle and label it $\angle D$. Then construct the angle bisector and label it $\overrightarrow{DM}$.

3. **You Decide?** Josh says that you get two obtuse angles after bisecting an angle. Brandon disagrees. Who is correct, and why?

Guided Practice

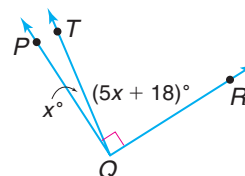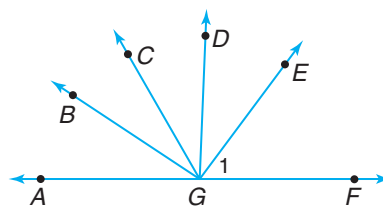**Getting Ready** **Use the Angle Addition Postulate to solve each of the following.**

Sample: If $m\angle 1 = 36$ and $m\angle 2 = 73$, find $m\angle 1 + m\angle 2$.

Solution: $m\angle 1 + m\angle 2 = 36 + 73$ or 109

4. If $m\angle 1 + m\angle 2 = 134$ and $m\angle 2 = 90$, find $m\angle 1$.
5. If $m\angle 1 + m\angle 2 = 158$ and $m\angle 1 = m\angle 2$, find $m\angle 1$.
6. If $m\angle 1 + m\angle 2 = 5x$ and $m\angle 1 = 2x + 1$, find $m\angle 2$.

Refer to the figure at the right.

7. If $m\angle AGB = 40$ and $m\angle BGC = 24$, find $m\angle AGC$. *(Example 1)*

8. If $m\angle BGD = 52$ and $m\angle BGC = 24$, find $m\angle CGD$. *(Example 2)*

9. If $\overrightarrow{GE}$ bisects $\angle CGF$ and $m\angle CGF = 116$, find $m\angle 1$. *(Example 4)*

10. **Algebra** Find $m\angle PQT$ and $m\angle TQR$ if $m\angle PQT = x$, $m\angle TQR = 5x + 18$, and $m\angle PQR = 90$. *(Example 3)*
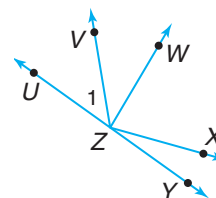
Exercises

Practice

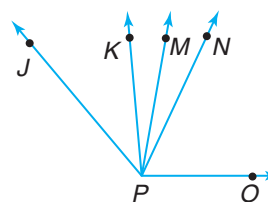**Refer to the figures at the right.**

11. If $m\angle UZW = 77$ and $m\angle VZW = 35$, find $m\angle 1$.

12. Find $m\angle VZX$ if $m\angle VZW = 35$ and $m\angle WZX = 78$.

13. If $m\angle WZX = 78$ and $m\angle XZY = 25$, find $m\angle WZY$.

14. If $m\angle UZW = 76$ and $\overrightarrow{ZV}$ bisects $\angle UZW$, find $m\angle UZV$.

Exercises 11–14

15. Find $m\angle KPM$ if $\overrightarrow{PM}$ bisects $\angle KPN$ and $m\angle KPN = 30$.

16. If $m\angle JPM = 48$ and $m\angle KPM = 15$, find $m\angle JPK$.

17. If $m\angle JPO = 126$ and $\overrightarrow{PN}$ bisects $\angle JPO$, find $m\angle NPO$.

Exercises 15–17

Homework Help

For Exercises	See Examples
11–20, 23, 24	1–3
22–24	4

Extra Practice
See page 730.

CONTENTS

Refer to the figure at the right.

Exercises 18–20

18. If $m\angle QSU = 38$ and $m\angle UST = 18$, find $m\angle QST$.

19. If RST is a right angle and $m\angle UST = 18$, find $m\angle RSU$.

20. Find $m\angle QSV$ if $m\angle TSU = 18$, $m\angle TSV = 24$, and $m\angle QSU = 38$.

21. If an acute angle is bisected, what type of angles are formed?

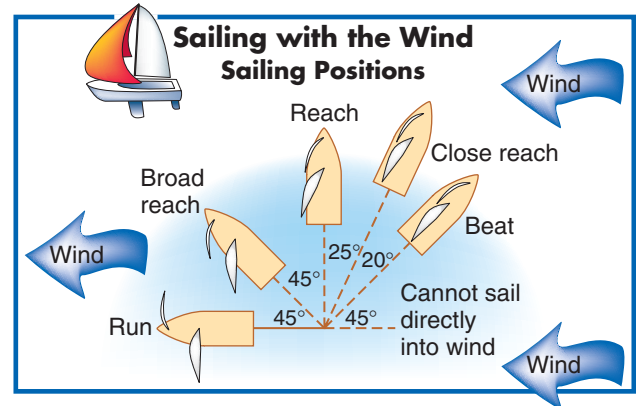**22.** What type of angles are formed when an obtuse angle is bisected?
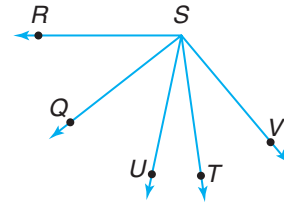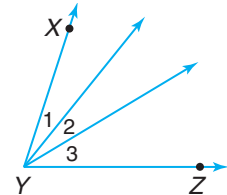
Exercise 23

Applications and Problem Solving

Real World

23. **Algebra**   If $m\angle 1 = 21$, $m\angle 2 = 5x$, $m\angle 3 = 7x + 3$, and $m\angle XYZ = 18x$, find x.

24. **Sailing**   The graph shows sailing positions. Suppose a sailboat is in the run position. How many degrees must the sailboat be turned so that it is in the close reach position?


Source: Coast Guard

25. **Critical Thinking**   What definition involving segments and points is similar to the Angle Addition Postulate?

Mixed Review

26. Use a protractor to measure $\angle ABC$.   *(Lesson 3–2)*

27. Name all angles having P as their vertex.   *(Lesson 3–1)*
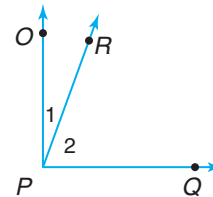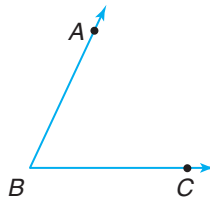
28. Points A, B, and C are collinear. If $AB = 12$, $BC = 37$, and $AC = 25$, determine which point is between the other two.   *(Lesson 2–2)*

Exercise 29

Standardized Test Practice
Ⓐ Ⓑ Ⓒ Ⓓ
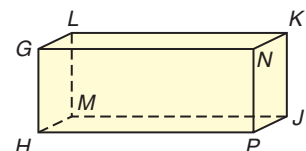
29. **Short Response**   Name the intersection of plane GNK and plane PJK.   *(Lesson 1–3)*

30. **Multiple Choice**   A stock rose in price from \$2.50 to \$2.75 a share. Find the percent of increase in the price of the stock. *(Percent Review)*

A 10%   **B** 9%   **C** 0.1%   **D** 0.09%